PHYS 331 – Assignment #4

Due Monday, December 4 at 08:00

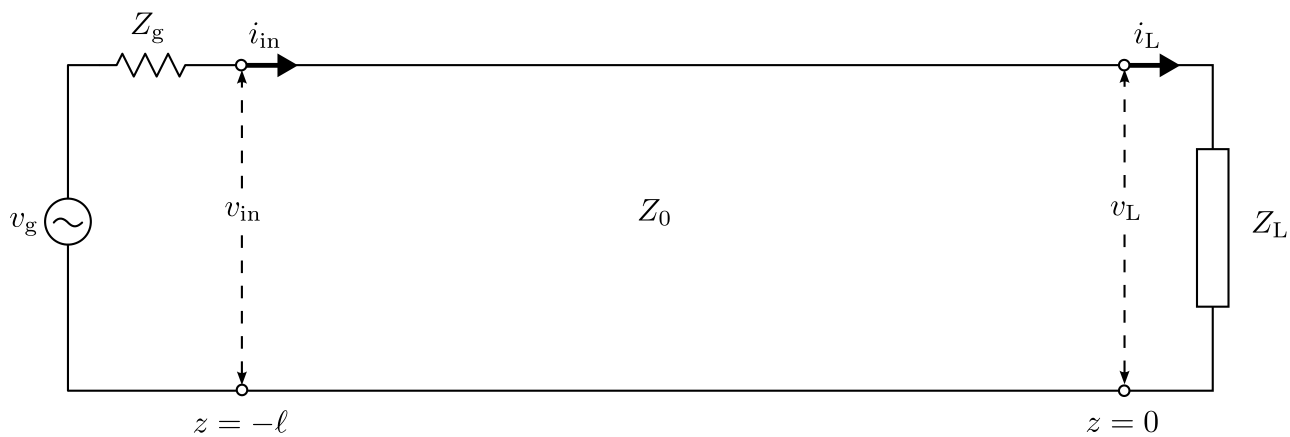

Figure 1: A signal generator with output impedance Z_g drives a transmission line of length ℓ that is terminated with load impedance Z_L.

Figure 1 shows the geometry of the transmission line system that we will use for all parts of this assignment. Assume $\ell = 8\,\text{m}$, $Z_0 = 50\,\Omega$, and that the signal propagation speed is $s = 0.7c$, where c is the vacuum speed of the light. In our discussion of transmission lines we showed that, in the frequency domain:

$$\hat{v}(z,\omega) = V_+ \left[e^{-j\omega z/s} + \Gamma e^{j\omega z/s} \right], \tag{1}$$

$$\hat{i}(z,\omega) = \frac{V_+}{Z_0} \left[e^{-j\omega z/s} - \Gamma e^{j\omega z/s} \right]. \tag{2}$$

1. For all parts of this first problem, we will assume that the output impedance of the signal generator is matched to the characteristic impedance of the transmission line such that $Z_g = Z_0$. Also assume that the output of the signal generator v_g is a square pulse with a height of $1\,\text{V}$ and width $25\,\text{ns}$. In class, we showed that, in this case:

$$V_+ = \frac{\hat{v}_g}{2} e^{-j\omega\ell/s}. \tag{3}$$

We also showed that, in the time domain, the voltage at the input of the transmission line ($z = -\ell$) is given by:

$$v_{\rm in} = \frac{v_{\rm g}(t)}{2} + \frac{\Gamma}{2} v_{\rm g}(t - 2\ell/s). \tag{4}$$

(a) Calculate the current at $i_{\rm in}(t)$ at the input of the transmission line. For this calculation, assume that Γ is independent of ω.

Plot $i_{\rm in}$ as a function of time for the cases $Z_{\rm L} = 0$, $50\,\Omega$, and ∞. You can sketch the current as a function of time by hand or you can plot it in Python using the linked code as a guide. If you sketch your plot by hand, you must include accurate and labeled scales for both the x- and y-axes.

(b) Calculate the voltage at $v_{\rm L}(t)$ at the load impedance. For this calculation, assume that Γ is independent of ω.

Plot $v_{\rm L}$ as a function of time for the cases $Z_{\rm L} = 0$, $50\,\Omega$, and ∞. You can sketch the voltage as a function of time by hand or you can plot it in Python using the linked code as a guide. If you sketch your plot by hand, you must include accurate and labeled scales for both the x- and y-axes.

(c) Calculate the current at $i_{\rm L}(t)$ at the load impedance. For this calculation, assume that Γ is independent of ω.

Plot $i_{\rm L}$ as a function of time for the cases $Z_{\rm L} = 0$, $50\,\Omega$, and ∞. You can sketch the current as a function of time by hand or you can plot it in Python using the linked code as a guide. If you sketch your plot by hand, you must include accurate and labeled scales for both the x- and y-axes.

For this problem, show the details of the calculations of $i_{\rm in}$, $v_{\rm L}$, and $i_{\rm L}$. Writing down only the correct answer will not result in full marks.